Iso-curvature fluctuations through axion trapping by cosmic string wakes

Biswanath Layek *
Institute of Physics, Sachivalaya Marg, Bhubaneswar 751005, India

Abstract

We consider wake-like density fluctuations produced by cosmic strings at the quark-hadron transition in the early universe. We show that low momentum axions which are produced through the radiation from the axionic string at an earlier stage, may get trapped inside these wakes due to delayed hadronization in these overdense regions. As the interfaces, bordering the wakes, collapse, the axions pick-up momentum from the walls and finally leave the wake regions. These axions thus can produce large scale iso-curvature fluctuations. We have calculated the detailed profile of these axionic density fluctuations and discuss its astrophysical consequences.

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*e-mail: layek@iopb.res.in

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I. INTRODUCTION

If QCD phase phase transition is of first order in nature then there are many important consequences in the context of early universe as well as in heavy ion collisions. In the conventional picture of a first order phase transition, the transition happens through homogeneous nucleation of hadronic bubbles in the background of QGP phases. Hadronic bubbles thus formed will keep expanding, coalesce and eventually complete the phase transition. One of the important consequences in this scenario, in the context of the early universe, is the concentration of baryon numbers inside the shrinking QGP bubbles which eventually can even form quark nuggets [1] due to high concentrations of baryon number inside the shrinking QGP phases. It was also shown that concentration of baryon number in the QGP phase can produce large baryon fluctuations which can survive against various dissipative processes [2] upto nucleosynthesis epoch and can alter the primordial abundances [3,4]. Ignatius and Schwarz studied [5] the presence of density fluctuations (those arising from inflation) at quark-hadron transition and showed that it will lead to splitting of the region in hot and cold regions with cold regions converting to hadronic phase first. Baryons will then get trapped in the (initially) hotter regions. Estimates of sizes and separations of such density fluctuations were made in ref. [5] using COBE measurements of the temperature fluctuations in CMBR. In our earlier work [6,7], we considered the effect of cosmic string induced density fluctuations on quark-hadron transition and showed that such density fluctuations at QCD epoch can lead to formation of extended planar regions of baryon inhomogeneity. Detailed profile of baryon inhomogeneities and it's effect on standard big bang nucleosynthesis were discussed in ref. [7].

Here, we study another aspect of the QCD phase transition in the context of early universe if the transition is of 1st order in nature, that is, axion inhomogeneity generation during quark-hadron transition in the presence of cosmic string induced density fluctuations. Axion was first introduced to solve strong CP problem [8] and has been extensively investigated primarily due to its possible role as a dark matter candidate. Recently it has been discussed [9] that due to the larger mass of axion in the hadronic phase compared to the QGP phase, axions will get trapped inside the shrinking QGP bubbles at the quark-hadron transition (somewhat in the similar manner as baryon trapping occurs in shrinking QGP bubbles [1]). The axions which are initially trapped inside the QGP phase will gradually pick up momentum from the walls of the shrinking bubbles and leave the QGP phase [9]. These axions which are left behind as the QGP bubbles collapse can form iso-curvature fluctuations. The minimum momentum p_{min} required to leave the QGP regions is given by,

$$p_{min} = \sqrt{(m_{ah}^2 - m_{aq}^2)} \equiv \Delta m \tag{1}$$

Where, m_{ah} and m_{aq} are the axion masses in the hadron phase and QGP phase respectively.

We study this phenomenon of axion trapping when cosmic strings are present in the early universe. Cosmic strings are generically produced by GUT phase transitions. Further,

it has been recently shown that superstring theories also lead to cosmic strings with acceptable parameters [10](see also ref. [11]). It therefore becomes important to investigate how such cosmic strings can affect the universe. (It is clear by now from latest WMAP data that contributions of cosmic strings and other topological defects to structure formation is insignificant [12]). We have shown earlier that the process of hadronization will be delayed in the cosmic string wakes at the quark-hadron transition [6,7]. The mechanism of shrinking quark phases is very different in our model compared to the conventional scenario based on homogeneous bubble nucleation. Geometry of the collapsing interface in our case is of sheet like planar structure unlike spherical in the conventional case. Thus, the iso-curvature fluctuations produced, as the axions leave the wake region, will spread in a sheet like regions in contrast to the spherical clumps as studied by Hindmarsh [9]. It will be interesting to study the subsequent evolution of large fluctuations with such geometrical structure and its astrophysical consequences. The paper is organized in the following manner. In sec. II, we briefly review the axion production from cosmic axionic string and it's temperature dependent mass following ref. [9]. Sec. III discusses the nature of density fluctuations as expected from straight cosmic string moving through relativistic fluid. Density fluctuations produced by the moving "wiggly" string is also discussed in this section. In Sec. IV, the dynamics of quark-hadron transition in the presence of string wakes and the mechanism of axion trapping in our model have been discussed. In Sec. V, we give our result. Summary and possible astrophysical consequences have been discussed in the final section VI.

II. AXION PRODUCTION FROM COSMIC STRINGS

The presence of the so called θ term in the QCD Lagrangian violates CP invariance. The most stringent bound on this $\theta(< 10^{-9})$ comes from the electric dipole moment measurement of neutron. Such extremely small value of θ in the strong interaction theory is called "strong CP problem". One of the most elegant solutions of this problem was first proposed by Peccei and Quinn [8] by introducing an axial $U(1)_{PQ}$ symmetry. The spontaneous breaking of this symmetry at some scale η_a can cause θ to settle down at some vacuum value. At the QCD scale instanton effects lead to explicit breaking of this $U(1)_{PQ}$ symmetry forcing θ to settle down at the true vacuum $\theta = 0$. However, due to spontaneous breaking of $U(1)_{PQ}$ symmetry, the phase of the field can wind around the vacuum manifold non-trivially in the physical space, forming the so called axionic string. After formation the strings are stuck in the plasma and are stretched by the Hubble expansion. During this time the dominant mechanism of dissipating energy is via heat due to the large frictional force exerted by the background plasma. However, with time the plasma becomes dilute and below a temperature T_* which is given by [13],

$$T_* = 2 \times 10^7 GeV (\frac{\eta_a}{10^{12} GeV})^2 \tag{2}$$

the strings move freely. The corresponding time t_*, can be obtained from the following time-temperature relation,

$$T^2 = \frac{0.3}{\sqrt{g_*}} \frac{m_{pl}}{t} \tag{3}$$

and is given by,

$$t_* = \frac{0.3}{\sqrt{g_*}} \frac{m_{pl}}{T_*^2} \simeq 1.8 \times 10^{-21} sec (\frac{10^{12} GeV}{\eta_a})^4 \tag{4}$$

Here $g_* = 106.75$ is the number of degrees of freedom relevant at temperature T_* (taking all the species in the standard model). After t_*, the string will lose energy through radiation of (pseudo) Goldstone bosons which are called axions. The axion is not truly a Goldstone Bosons. It will acquire mass due to the axial anomaly [14] once the instanton effect turns on. In the high temperature phase ($T > T_c$), the mass of axions arises from the θ-dependent part of the free energy density which have been calculated by Preskil et al. [15] in the dilute-instanton- gas approximations. For three quark flavors appropriate for T not too high compared to T_c, they have determined the mass as,

$$m_{aq}(T) \simeq 2 \times 10^{-2} [\frac{\Lambda^2}{\eta_a}][\frac{m_u m_d m_s}{\Lambda^3}^{1/2}][\frac{\Lambda}{\pi T}]^4 [9 ln \frac{\Lambda}{\pi T}]^4 \tag{5}$$

Where, m_u, m_d, m_s are the current quark masses of u, d and s quark respectively and $\Lambda \equiv \Lambda_{QCD} \sim 200 MeV$ is the QCD scale. Taking critical temperature $T_c \sim 150 MeV$ and putting the values of u, d and s quark masses (see ref. [9] for details), we get the axion mass at the onset of QCD transition and can be written in terms of axionic string formation scale as,

$$m_{aq}(T_c) = 5.3 \times 10^{-7} eV (\frac{10^{12} GeV}{\eta_a}) \tag{6}$$

At low temperature in the hadron phase, the axion mass is calculated from the current-algebra technique which gives the value [15],

$$m_{ah} = \frac{(m_u m_d)^{1/2}}{m_u + m_d} \frac{m_\pi f_\pi}{\eta_a} = 6.04 \times 10^{-6} eV (\frac{10^{12} GeV}{\eta_a}) \tag{7}$$

Here, m_π and f_π are the mass and decay constant of pions respectively. The axion mass being temperature dependent at high temperature phase as in Eq.(5), it's very light and ultra-relativistic at very high temperature. It becomes dynamically significant at the temperature at which compton wave length of the axions fall within the horizon. This is the time ($\tilde{t}$), at which axion string will not be able to oscillate sufficiently to radiate axions due to damping of domain walls attached to each string. From the time temperature relation Eq.(3) one can get the time $\tilde{t}$, from the following relation (neglecting the logarithmic dependence on T in Eq.(5)),

$$m_{aq}(\tilde{t}) = m_{aq}(t_c)(\frac{\tilde{t}}{t_c})^2 = d_h(\tilde{t}) = (2\tilde{t})^{-1} \tag{8}$$

Where, $m_{aq}(t_c)$ is the mass of the axion at the onset of QCD phase transition obtained from Eq.(5). Thus, one gets the time $\tilde{t}$ as,

$$\tilde{t} = \frac{1}{2}[m_{aq}(t_c) t_c]^{-1/3} t_c \simeq 1.6 \times 10^{-3} sec (\frac{\eta_a}{10^{12} GeV})^{1/3} (\frac{t_c}{sec})^{2/3} \tag{9}$$

Here, we will study the fluctuations in the density of axions which are produced through the radiation from the axionic string during the era, $t_* < t < \tilde{t}$ as in ref. [9]. During this time one can assume that string network will enter into the scaling regime which has been discussed extensively in literature (see, ref. [16]). Under the scaling assumption the spectrum of axions in the comoving momentum range k^* and $\tilde{k}$ can be considered to be flat [9]. We will take the number density distribution of axions resulting from axionic string decay in the comoving momentum range $k^*(=\frac{(R(t_*)}{t_*})$ and $\tilde{k}(=\frac{R(\tilde{t})}{\tilde{t}})$ as given in ref. [9] as,

$$n_k dk = R(\tilde{t}) G \eta_a^2 ln(\eta_a \tilde{t}) \rho_c dk/k^2 \tag{10}$$

Where, $R(\tilde{t})$ is the scale factor at time $\tilde{t}$, and ρ_c is the corresponding critical density of the universe. We will consider the above density distribution with suitable dilution factor due to expansion of the universe inside the wake-like overdensity density regions produced at QCD epoch as discussed in the following section.

III. DENSITY FLUCTUATIONS ARISING DUE TO LONG COSMIC STRINGS

In this section we will review the formation of density fluctuations due to cosmic string moving through relativistic fluid. First, we will discuss the case for a long straight string assuming it does not have any small scale structure. Though, simulations [17] on the string network in the expanding universe show that the long cosmic string possesses substantial amount of small scale "wiggly" structures running along the strings. The presence of such wiggles makes the string heavier and slows down the motion of the string as a whole. However, as we discuss below, even the presence of such wiggles along the string will not alter the order of magnitude estimate of density fluctuations produced at QCD epoch. Therefore, for simplicity of presentation we will consider the case of straight (i.e., with no small scale structure) strings first and then we briefly discuss for the case of wiggly strings.

To discuss the generation of density fluctuations by moving straight string, we take the metric describing the space-time around the string (lying along z-axis) as [18],

$$ds^2 = dt^2 - dz^2 - dr^2 - (1 - 4G\mu)^2 r^2 d\psi^2. \tag{11}$$

Here, μ is the mass per unit length (equal to string tension) of the straight string. This metric can be put in Minskowski form by defining angle $\psi' = (1-4G\mu)\psi$, where new angle ψ' varies between 0 and $(1-4G\mu)2\pi$ instead of 0 and 2π. A test particle at rest with respect to the string does not feel any gravitational force. However, when the string moves with velocity v_{st}, the nearby matter gets an impulsive velocity along the direction of the surface swept by the moving string. For collisionless particles, the resulting impulsive velocity [16] is $v_{impul} = 4\pi G\mu v_{st}\gamma_{st}$. This effect is responsible for the formation of wake [19] and for collisionless particles, this leads to planar structure of density fluctuation. In this case, the density fluctuations will be of order unity with opening angle of the wake being equal to the deficit angle $8\pi G\mu$. However, at QCD epoch the universe was dominated by the relativistic plasma. So, to study the density structure of the wake at that epoch, the description of matter should be taken in terms of a relativistic fluid rather than collisionless particles. Generation of density fluctuations due to a cosmic string moving through a relativistic fluid has been

analyzed in the literature [20–22], where it has been shown that for supersonic velocities of string, a wake-like shock is formed behind the string. We will follow ref. [22], where the equations of motion of a relativistic fluid are solved in the string space-time (Eq.(11)), and both subsonic and supersonic flows are analyzed. One finds that for supersonic flow, a shock develops behind the string, just as in the study of ref. [20,21]. Following ref. [22], the resulting density fluctuations can be expressed in terms of four-velocity and given by,

$$\frac{\delta\rho}{\rho} \simeq \frac{16\pi G\mu u_f^2(1+u_s^2)}{3u_s\sqrt{u_f^2-u_s^2}}, \qquad sin\theta_w \simeq \frac{u_s}{u_f}, \tag{12}$$

where $u_f (= v_f/\sqrt{1-v_f^2})$ and $u_s (= v_s/\sqrt{1-v_s^2})$ are the four velocity of fluid (in the string rest frame) and four velocity of sound respectively, with v_s being the sound speed. θ_w is the angle of the shock. In this case, when string velocity v_f is ultra-relativistic, then one can get strong over densities (of order 1) and the angle of the wake approaches the deficit angle $\simeq 8\pi G\mu$. To discuss the effect of density fluctuations on QCD phase transition we will follow our previous work [6,7] and take the sample value of density fluctuations and the angle of the shock as,

$$\theta_w \simeq 20^0, \qquad \delta\rho/\rho \simeq 3\times 10^{-5}. \tag{13}$$

These values are obtained from Eq.(12) for string velocity of 0.9, sound velocity $v_s = 1/\sqrt{3}$ and $G\mu \sim 10^{-6}$ (for 10^{16} GeV GUT strings).

Here, we should mention that the axion being very weakly interactive [9], they can be considered to be a fluid consisting of collisionless particles. Therefore, description of wake formation for collisionless particles as mentioned above can be operative for axions also and can lead to formation of axionic density fluctuations $(\frac{\delta\rho}{\rho})_{axion}$ of order unity with angle of the wake, $\theta_w \sim 8\pi G\mu$. These over dense axions could be concentrated within very thin sheet like region of thickness $8\pi G\mu\ d_H \sim 1cm$ ($d_H \sim$ 10 km being the horizon size at $T_c = 150$ MeV). (One has to properly account for the wavelength of axions.) However, we will see below that the magnitude of the density fluctuations produced through axion trapping (which is our main topic of discussion in this paper) by the collapsing interfaces will be very large compared to the above density fluctuation. Essentially, this thin sheet like region will be trapped initially inside the wake of larger thickness which is produced by the formation of shock by moving string through relativistic fluid (as discussed above). Ultimately, axions will also escape from this thin wake and contribute in producing overall iso-curvature fluctuations. However, their effect will be very insignificant. Even the presence of wiggles along the string as we discuss below, will not change the order of magnitude estimate of such axionic density fluctuations. Therefore, to make the physical picture of axion trapping simpler, we will not consider these kind of fluctuations in this work and we will focus only on iso-curvature fluctuations produced through trapping of axions inside the collapsing interfaces of the wake.

So far we have discussed density fluctuations produced by the straight string assuming it does not have any small scale structure. But, as we mentioned above, the string has small-scale wiggly structures [17] running along the string. The presence of such wiggles increase the average energy density of the string and makes the strings to move slower. For

physics at large scales (compared to the scale of wiggles l_w, which is expected to be set by the gravitational-radiation from the string [23] and given as, $l_w \simeq \Gamma G\mu t$ where $\Gamma \sim 100$), the effect of wiggles is taken into account by defining the effective mass per unit length μ_w and the string tension T_w. Unlike the case of straight string, these quantities for perturbed string are not equal and they are related by the equation of state [23], $\mu_w T_w = \mu^2$. Due to the presence of these small scale structure, the static wiggly strings develop Newtonian gravitational field and a test particle at rest with respect to such perturbed string experiences attractive gravitational force. The metric around the wiggly string and density structure of the wake for collisionless particles have been studied and discussed in the literature [16,23]. It turns out that, for moving wiggly string, total impulsive velocity imparted on the particles towards the surface swept out by the string can be written as [23],

$$v_{impul} = 4\pi G\mu_w v_{st}\gamma_{st} + \frac{2\pi G(\mu_w - T_w)}{v_{st}\gamma_{st}}. \tag{14}$$

Where, first term is the usual velocity impulse arises due to the conical structure of the space-time around the wiggly string, with μ being replaced by the effective mass per unit length μ_w. The second term arises due to the Newtonian gravitational field developed by the wiggliness of the string. Using the value of $\mu_w \sim 1.9\mu$ and $T_w \sim 0.5\mu$ (see ref. [16]), we see from Eq.(14) that if the string moves with ultrarelativistic speed, then the first term will dominate over the gravitational term. The rms velocity of the wiggly string on the scale of smallest wiggles found out to be, [24] $v_{st} \sim 0.6$. For this case, gravitational term can be neglected. In this case, density fluctuations for collisionless particles will be order of unity with opening angle, $\theta_w \simeq 8\pi G\mu_w$. However, the velocity of the string obtained by taking average over a correlation length comes out to be very small ($v_{st} \sim 0.15$) for which gravitational effect should be taken into account. Formation of density structure of wake for collisionless particles have been studied in ref. [23] assuming the particles remain at a distance far from the string and the particles will experience only averaged effect of the wiggles (in this case, relevant velocity is, $v_{st} \sim 0.15$). In the string rest frame (string lying along z-axis), if the particles moves with velocity v_{st} in the (+ve) x direction, then following ref. [23], resulting density fluctuations can be written as,

$$(\frac{\delta\rho}{\rho})_{axion} \simeq 1 + 4G(\mu_w - T_w)(\frac{1 - v_{st}^2}{v_{st}^2})(\frac{x - X_0}{X_0}) \tag{15}$$

Where, X_0 is of order of the inter-string separation. Note, setting the second term equal to zero, one recovers the result for density structure of the wake for straight string. If we put the value of wiggly string parameter in Eq.(15), we see that order of magnitude of density fluctuations will not be affected compared to the case of straight string. We will see later that due to the trapping mechanism, axion density can increase in the wake by many orders of magnitude. We thus conclude that generation of density fluctuation by axions (being collisionless) by above phenomenon is insignificant compared to the case where axionic fluctuations happens through trapping mechanism inside the wake produced by relativistic fluid. Therefore, we will ignore this effect in our subsequent discussion.

Now, we discuss the effect of wiggliness on the value of density fluctuations (Eq.(13)) produced by moving string through the relativistic fluid. Study on formation of shock

by moving wiggly string has been done in the work of ref. [23] under some simplifying assumptions. In their study, authors have treated the fluid as a non-viscous, non-relativistic fluid. As has been argued in ref. [23], the assumptions of non-relativistic fluid can be good approximation for the string velocity $v_{st} \sim 0.2$ which is relevant for the distance scale larger than the scale of the wiggles. Formation of wakes for wiggly strings has been discussed in ref. [23] after recombination. It is also mentioned in ref. [23] that similar wake should arise due to wiggly string even at earlier stages when string moves through the relativistic plasma, even with $v_{st} = 0.15$ for the wiggly string. If we follow the approach of ref. [20–22], then $v_{st} = 0.15$ is subsonic and no shock can form. However, this does not represent the actual physical situation. This is because at smaller distance scales (compared to the scale of the wiggles), the straight segments of the string move with $v_{st} \simeq 0.6$ (ref. [24]). This is supersonic motion and shock will form. Due to random velocities of different segments of the string, the individual shocks will combine to give some resultant shock and hence, wake for the total length of the (wiggly) string. Thus we will assume that even with the wiggles, shock and wake formation results. For these wakes we will use Eq.(12) with $v_{st} = 0.6$ and mass per unit length μ for the straight string (as individual segments are straight). However, now the length scale of the shock (wake) will be governed by the size of each straight segment. The length of the wake (in the direction away from the string) will still be governed by the average inter-string separation. That will not be affected by the presence of the wiggles. (Recently, the evolution of wiggly string network has been studied [25] on flat space-time. The results suggest that even in the presence of wiggles the string network obey scaling solution. Authors in ref. [25] also mentioned that the average inter-string separation also scales with time. Though above simulation has been done on flat space-time, the author believes that such scaling solution is expected to exhibit in expanding universe also. If this is the case, then on an average inter-string separation for long wiggly string will not be altered from the straight string case.) However, the width of the wake (along the string direction) will now be truncated down to the average size of individual straight segment. Note however that, this simply changes the geometry of a planar wake (for a completely straight string) to a collection of strips with each strip corresponding to individual straight segment of the wiggly string. Even the geometry of these strips may not be planar due to rapidly changing velocity of string segments. For a correct treatment, one should properly account for the different velocity and orientations of these strips, however, for present we will ignore these complications. Following Eq.(12), the opening angle of the wake and density fluctuation for each straight segment of the wiggly string can now be written as (using $v_{st} = 0.6$ and sound velocity $v_s = 1/\sqrt{3}$),

$$\theta_w \simeq 70^0, \qquad \delta\rho/\rho \simeq 4 \times 10^{-5}. \tag{16}$$

As the individual shock will combine to give resultant shock, we expect that the overall volume of the wakes will remain same (upto some factor) as for straight string case. Here, we should again emphasizing that the above value of density fluctuation and opening angle of the wake (i.e., Eq.(16)) is derived from Eq.(12), where it is assumed that the fluid flow to be uniform. In reality, due to the wiggliness of the string the fluid will experience the rapidly changing directions of the wiggles which causes acceleration of the fluid in different direction. So, in proper treatment of shock formation by wiggly string, one should take the

non-uniformity nature of the fluid in the analysis. However, due to lack of such complicated proper analysis, we will use (mainly) Eq. (12) and Eq.(16) and discuss the effect of such fluctuations on QCD phase transition.

IV. EFFECT OF STRING WAKES ON QUARK-HADRON TRANSITION AND AXION TRAPPING MECHANISM IN OUR MODEL

Here, we will first briefly discuss the dynamics of QCD phase transition in the context of early universe. In the conventional scenario of first order phase transition, the transition happens through the nucleation of hadronic bubble in the QGP background. These bubbles then grow, coalesce and finally convert the whole QGP phase into hadronic phase. However, since the critical size of the bubble beyond which it can grow is quite large at the temperature closed to T_c and also the nucleation rate too small at such temperatures, universe has to supercool to a temperature T_{sc} to begin the nucleation process. The amount of supercooling ΔT_{sc} depends on the surface tension σ and the latent heat L. We take sample values of these parameters as obtained from the lattice results [26], $\sigma \simeq 0.015 T_c^3$ and $L \simeq 3T_c^4$. As we discussed in ref. [7], for smaller value of surface tension our results remain applicable as the supercooling would be smaller (as has been argued in the literature [27], see also ref. [28].) and cosmic string induced density fluctuations will have more prominent effect on the dynamics of quark-hadron transition. Taking the values of these parameters one can estimate the amount of supercooling to be [5,29]

$$\Delta T_{sc} \equiv 1 - \frac{T_{sc}}{T_c} \simeq 10^{-4} \tag{17}$$

for critical temperature $T_c = 150$ MeV.

As soon as the universe supercool down to temperature T_{sc}, hadronic bubbles will start nucleating in the background of quark gluon plasma phases. The duration of this nucleation process is very small and lasts only for a temperature range of $\Delta T_n = (1 - \frac{T_n}{T_c}) \simeq 10^{-6}$, for a time duration of order $\Delta t_n \simeq 10^{-5} t_H$ (t_H is the Hubble time). The bubbles which have already been nucleated will keep expanding and it will release latent heat which will prevent further nucleation process. After the nucleation processes stop, the universe enters into the so called slow combustion phases [1]. However, in our model [6,7] this slow combustion phase is very different from the standard scenario. Here the phase transitions happens in the presence of density fluctuations (which will transform into temperature fluctuations) which is produced as the string passes through the relativistic plasma. In this scenario, as we discuss below, the hadronization inside the wake will be delayed while outside the wake the universe will enter into slow combustion phases. To understand this slow combustion phase in our model we take density fluctuation from Eq.(13), which will transform into temperature fluctuation of magnitude $\Delta T_{wake} \equiv \delta T/T \simeq 10^{-5}$. Since, this temperature fluctuation is larger than ΔT_n, there will be no nucleation inside the wake while nucleation process will get completed outside the wake. Thus, the region outside the wake will enter into slow combustion phase, while overdensity region will till remain in QGP phase. For this, the overdensity in the wake should not decrease in the time scale Δt_n. To calculate

the time scale t_{shk} of the evolution of the overdensity we take the typical average thickness of the wake as,

$$d_{shk} \simeq sin\theta_w \ d_H \simeq 3km. \tag{18}$$

Where, $d_H \simeq 10$ km is the horizon size at $T_c = 150$ MeV. (Note, as we have discussed earlier, the shock parameters of Eq.(16) apply only for a straight segment of the string. However, the resulting wake may still extend upto typical inter-string separation. Thus the overall volume covered by the wakes of different segments of wiggly string could be of same order as that of a wake from a straight string. We take this to be the case.) Now, typical time scale for the evolution of the overdensity will be governed by the sound speed v_s. We take $v_s \simeq 0.1$ suitable at temperature closed to T_c (see ref. [5,30]). Thus, we get the time scale as,

$$t_{shk} \simeq \frac{d_{shk}}{v_s} \simeq 20 \ sin\theta_w \ t_H \tag{19}$$

which is large compared to the duration of the nucleation process $\Delta t_n \simeq 10^{-5} t_H$. It is also much larger than $\Delta t_{trnsn} (\simeq 14 \mu sec)$ during which the quark phase is completely converted to the hadronic phase in the region outside the wake. Thus, one can conclude that the region outside the wake enters the slow combustion phase before any significant bubble nucleation can take place in the wake region. Heat released by the bubbles outside the wakes will prevent bubble nucleation everywhere. It is then possible that the region outside the wakes will be converted to the hadronic phase while inside the wakes QGP phase remains. Further completion of the phase transition will happen when the interfaces, seperating the QGP phase inside the wakes from the outside hadronic regions, move inward from the wake boundaries. These moving, macroscopic, interfaces may trap most of the baryon number in the entire region of the wake (and some neighborhood) towards the inner region of the wake. Finally the interfaces will merge, completing the phase transition, and leading to a sheet of very large baryon number density, extending across the horizon. Detailed profile of baryon inhomogeneities produced through these collapsing interface had been estimated in our previous work [7].

Here, we want to discuss the mechanism of axions which will get trapped initially inside the wake and subsequently leave after acquiring required momentum form the collapsing walls. For that we will concentrate on a single wake like overdensity region. Typical average volume of such region will be governed by the total number of long strings in a given horizon. From numerical simulation [24], the number of long strings is expected to be about 15. (As is mentioned earlier, simulation [25] on wiggly string network suggests that even in the presence of wiggles on long string, the network obey scaling solution. Though above simulation has been done on flat space-time, the author believes that such scaling solution is expected to exhibit in expanding universe also. If this is the case, then the number of long strings as we have taken for straight string case will remain applicable even for wiggly string also.) If the string wakes are reasonably parallel, then they will span most of the horizon volume, as the average thickness of a wake will be order of 1-2 km. In such a situation, the hadronic phase will first appear in the regions between the wakes, which may cover a very small fraction of the horizon volume initially. If f be the volume fraction occupied by the QGP phase then

the initial value of this fraction f will be close to 1. f will then slowly decrease to zero as the planar interfaces (formed by the coalescence of bubbles in the region in between the overdense wakes) move inward, converting the QGP region inside the wake into the hadronic phase. Certainly, the actual situation will be more complicated than this, with string wakes extending in random directions, and often even overlapping. In such a situation, even the initial value of f, when bubble coalescence (in the regions between the wakes) forming planar interfaces, may be smaller than 1. Note that, for wiggly string case, the wakes formed by straight segments of the string will have different orientations depending on the direction of each straight segment. In this situation, distribution of string wakes will be even more complicated, hence in determining the exact initial value of f. However, for simplicity, we will take the initial value of f to be almost 1 as is done in ref. [7] and focus on the region relevant for only one string, covering about 1/15 of the horizon volume. Also, as our main focus in this work is an order of magnitude estimate of iso-curvature fluctuations and since underline physical picture will remain same even for wiggly string, hence from now onwards, we only consider the case of straight string and the results will be quoted only for straight string case.

We now study the density evolution of axions which may get trapped initially inside the wake. For that, let us first recall the effect of expansion of the universe on the dynamics of the phase transition. For that we mostly follow the work of Fuller [2] (see also, [6,7]) who have studied the density evolution of baryons inside the shrinking QGP bubbles. If $R(t)$ is the scale factor of Robertson-Walker metric, then Einstein's equations give [2],

$$\frac{\dot{R}(t)}{R(t)} = \sqrt{\frac{8\pi G\rho(t)}{3}}, \tag{20}$$

where ρ is the average energy density of the mixed phase and given by,

$$\rho = f\rho_q + (1-f)\rho_h \tag{21}$$

The energy density, pressure (ρ_q, p_q) in the QGP phase and in the hadronic phase(ρ_h, p_h) are given by,

$$\rho_q = g_q a T^4 + B,\ p_q = \frac{g_q}{3} a T^4 - B \tag{22}$$

$$\rho_h = g_h a T^4,\ p_h = \frac{g_h}{3} a T^4. \tag{23}$$

Here $g_q \simeq 51$ and $g_h \simeq 17$ are the degrees of freedom relevant for the two phases respectively (taking two massless quark flavors in the QGP phase, and counting other light particles) [2] and $a = \frac{\pi^2}{30}$. At the transition temperature we have $p_q = p_h$ which relates T_c and the bag constant B as, $B = \frac{1}{3} a T_c^4 (g_q - g_h)$. We define $x = g_q/g_h$ to be the ratio of degrees of freedom between the two phases. With this, Eq.(20) can be written as,

$$\frac{\dot{R}(t)}{R(t)} = (\frac{8\pi G B}{3})^{1/2} [4f + \frac{3}{x-1}]^{\frac{1}{2}}. \tag{24}$$

Now, conservation of the energy-momentum tensor gives,

$$R(t)^3\frac{dp(t)}{dt} = \frac{d}{dt}\{R(t)^3[\rho(t)+p(t)]\}. \tag{25}$$

During the transition, T and p are approximately constant. Therefore, Eq.(25) can be rewritten as,

$$\frac{\dot{R}(t)}{R(t)} = -\frac{\dot{f}(x-1)}{3f(x-1)+3}. \tag{26}$$

These two equations, Eq.(24) and Eq.(26) have to be solved simultaneously to determine the evolution of scale factor and the change of f, the volume fraction of QGP region as the transition proceeds. The initial time (t_0) relevant for us is when the overdensity inside the wake has been formed. Taking 15 long string inside the horizon at that epoch, we will consider the initial volume as,

$$V_0 \approx (\frac{1}{15})d_H^3, \tag{27}$$

where $d_H(= 2t_0)$ is the size of the horizon at time t_0. Note that we take the wake like overdense regions to be well formed at time t_0. Thus our representative volume as the transition proceeds is, $V(t) = V_0(\frac{R(t)}{R_0})^3$, R_0 being the scale factor at t_0. Taking center of the wake as the origin and considering motion of the interface along z direction, we can write

$$f(t)V(t) = 2A(t)z(t). \tag{28}$$

Where, $A(t)$ is the area of each sheet. Assuming the sheet extending across the horizon, we get the area as a function of time as, $A(t) \sim V(t)^{2/3}$. Putting this value we get the evolution of the thickness $z(t)$ as,

$$z(t) = \frac{f(t)}{2}V_0^{(\frac{1}{3})}\frac{R(t)}{R_0}. \tag{29}$$

Note that we are approximating the wake as bounded by two parallel sheets separated by average thickness of the wedge. Now let us determine the effect of interface motion on axion momentum distribution and subsequently on the evolution of number density of axions inside the wake. Total number of axions which will remain initially inside the wake can be calculated by integrating Eq.(10). Thus, if $N(t_0)$ be the total number of axions trapped initially inside the wake, then,

$$N(t_0) = V_0\int_{k_{min}}^{k_{max}} n_k dk(\frac{R(\tilde{t})}{R_0})^3. \tag{30}$$

Where the last factor $(\frac{R(\tilde{t})}{R_0})^3$ is due to the decrease of axion density from time $\tilde{t}$ to t_0 causes by the expansion of the universe (n_k as given in Eq.(10) gives the number density of axions at time $\tilde{t}$). k_{min} and k_{max} are minimum and maximum comoving momentum respectively. Since, the minimum comoving momentum which can be trapped initially inside the wake cannot be smaller than $\frac{2\pi R_0}{z_0}$, (z_0 is the initial average thickness of the wake) we will take k_{min} as $\tilde{k}$ or $2\pi R_0/z_0$ whichever is larger. To determine that, we take the ratio,

$$\frac{\tilde{k}z_0}{2\pi R_0} = \frac{R(\tilde{t})}{\tilde{t}}\frac{z_0}{2\pi R_0} = (\frac{1}{\tilde{t}t_0})^{1/2}\frac{z_0}{2\pi} \tag{31}$$

Taking $\tilde{t}$ from Eq.(9), t_0 as $\sim 10^{-5}sec$ and the initial thickness $z_0 \sim$ 1km or so, we get the ratio of $O(1)$. So, we take the minimum comoving momentum k_{min} as $\tilde{k}$. The maximum comoving momentum k_{max} of the axions which will remain inside the wake can be set as discussed below.

The momentum of the axions will keep increasing with the collapsing interfaces [9]. Thus, if $k(t)$ be the comoving momentum of an axion whose initial momentum is k_i, then the momentum at any time t, when thickness of the wake becomes $z(t)$ can be obtained from the following relation,

$$k(t) = k_i\frac{z_0}{z(t)}\frac{R(t)}{R_0} \tag{32}$$

The minimum physical momentum required for the axions to leave the interface is equal to Δm (see, Eq.(1)). Therefore, the axions which have comoving momentum less than $\Delta m R_0$ will remain inside the wake. Thus, we can put the upper limit of the integration in Eq.(30) as, k^* or $\Delta m R_0$ whichever is smaller. To determine that let us again calculate the ratio,

$$\frac{k^*}{\Delta m R_0} = \frac{\frac{R(t^*)}{t^*}}{\Delta m R_0} = (\frac{1}{t^* t_0})^{1/2}\frac{1}{\Delta m} \tag{33}$$

Putting the value of Δm as obtained from Eq.(6) and Eq.(7) and the value of t_* from Eq.(4) the ratio can be determined in terms of the formation scale of axionic strings as,

$$\frac{k^*}{\Delta m R_0} \simeq 7.9(\frac{t_0}{sec})^{-1/2}(\frac{\eta_a}{10^{12}GeV})^3 \simeq 1.2\times 10^3(\frac{\eta_a}{10^{12}GeV})^3 \tag{34}$$

Thus, the above ratio depends on the formation scale η_a of the axionic string, which is constrained by the terrestrial and astrophysical experiments as well as from the cosmological considerations. The most stringent lower bound has been obtained from the SN 1987A [31] as $\eta_a \geq 10^{10}GeV$ and the upper bound [15] ($\eta_a \leq 10^{12}GeV$) comes from the consideration that the axions should not overclose the universe. If we take $\eta_a < 10^{11}GeV$, then the above ratio is always less than unity. In this case we can take the upper limit of integration in Eq.(30) as k^*. Having set the limits of integration we can now calculate total number of axions which will remain initially inside the wake and given by,

$$N(t_0) = V_0 F[\frac{1}{\tilde{k}} - \frac{1}{k^*}](\frac{R(\tilde{t})}{R_0})^3 \tag{35}$$

Where, $F = R(\tilde{t})G\eta_a^2 ln(\eta_a\tilde{t})\rho_c \simeq \frac{3}{32\pi}\eta_a^2 ln(\eta_a\tilde{t})(\tilde{t}t_0)^{-1/2}\frac{R_0}{\tilde{t}}$. Now as the interfaces move towards each other momentum of each axion will be modified according to Eq.(32). Let $n_{k'}dk'$ be the modified spectrum due to trapping of axions within the collapsing walls when thickness becomes $z(t)$. Unless the momentum of each axion becomes $\Delta m R(t)$ the axions will remain inside the wake. So upto certain time t_1, total number of axions $N(t)$ (for $t_0 < t < t_1$)

will remain fixed inside the wake. Subsequently, number density will be increased due to decrease of volume fraction of QGP region f caused by interface motion. So, total number of axions at certain time $t(< t_1)$ can be written as,

$$N(t) = fV(t)\int_{k'_{min}}^{k'_{max}} n'_k dk' = fV(t)\int_{k'_{min}}^{k'_{max}} F'\frac{1}{k'^2}dk' \quad (t_0 < t < t_1) \tag{36}$$

Where, F' is new coefficient will be determined in terms of F. This coefficient F' essentially will take care of the change of the coefficient F due to shifting of momentum of each axion as is obtained from Eq.(32) and the change in QGP volume causes by interface motion. Determining this coefficient in terms F will be particularly useful in determining the evolution of axion density when axions will start leaking out the overdensity regions. $k' = k\frac{z_0}{z(t)}\frac{R(t)}{R_0} \equiv kc(t)$ (say) and k'_{min}, k'_{max} have to be substituted by $\tilde{k}c(t)$ and $k^*c(t)$, respectively. Substituting all these quantities and integrating Eq.(36) we get the total number of axions at any intermediate time between t_0 to t_1. Equating this number to the initial total number of axions at t_0 as obtained from Eq.(35) one gets the coefficients F' as follows,

$$F' = \frac{V_0 F c^2(t)}{fV(t)}(\frac{R(\tilde{t})}{R_0})^3 \tag{37}$$

Putting back F' in Eq.(36) and divided by the QGP volume $fV(t)$ we get the number density of axions upto time t_1 and given by,

$$\rho(t) = \frac{N(t)}{fV(t)} = \frac{V_0 F}{fV(t)}(\frac{R(\tilde{t})}{R_0})^3[\frac{1}{\tilde{k}} - \frac{1}{k^*}] = \frac{3}{32\pi}\frac{V_0\eta_a^2}{fV(t)t_0^2}ln(\eta_a\tilde{t})[(\tilde{t}t_0)^{1/2} - (t_*t_0)^{1/2}] \tag{38}$$

Now, as we mentioned earlier that the density written above is applicable upto time t_1 below which no axions will leave the QGP regions due to insufficient momentum required to escape the region. The thickness of the wake at that time t_1 can be calculated using the fact that when the momentum of axion exceeds the value $\Delta m R(t)$ they will leave the wake region, which is first happens when $k^*c(t)$ becomes equal to $\Delta m R(t)$. Using this equality we get the corresponding thickness at t_1 and determined by,

$$k'_{max} \equiv c(t_1)k^* = \frac{z_0}{z(t)}\frac{R(t_1)}{R_0}\frac{R(t^*)}{t^*} = \Delta m R(t_1) \tag{39}$$

Or,

$$\frac{z(t_1)}{z_0} = \frac{1}{\Delta m}(t^* t_0)^{-1/2} \simeq 7.8(\frac{t_0}{sec})^{-1/2}(\frac{\eta_a}{10^{12}GeV})^3 \simeq 1.2 \times 10^{-3} \tag{40}$$

Since, the initial thickness z_0 of the wake is $\sim$ 1km, and $t_0 \sim 10^{-5}sec$, thickness of the wake at time t_1 comes out to be $z(t_1) \sim 2m$ for string formation scale $\eta_a \sim 10^{10}GeV$. As soon as the thickness decreases down to $\sim 2m$, the axions will start leaving the QGP region and finally all the axions will leave the wake and form a sheet like structure. To determine the evolution of density after t_1, upper and lower limit of the integration of Eq.(36) should be replaced by $\Delta m R(t)$ and $\tilde{k}c(t)$ respectively. Thus, the evolution of axion density within the wake after time t_1 is given by,

$$\rho(t) = \frac{V_0 F c(t)}{f V(t)} (\frac{R(\tilde{t})}{R(t_0)})^3 [\frac{1}{\tilde{k}c(t)} - \frac{1}{\Delta m R(t)}] \quad (41)$$

$$= \frac{3}{32\pi} \frac{V_0 \eta_a^2}{f V(t) t_0^2} ln(\eta_a \tilde{t}) [(\tilde{t} t_0)^{1/2} - \frac{z_0}{z(t)\Delta m}], (t_1 < t < t_f) \quad (42)$$

Where, Eq.(37) and Eq.(38) have been used. t_f is the final time at which all the axions will leave the QGP region. The corresponding thickness can be obtained from the following expression,

$$k'_{min} \equiv \tilde{k}c(t_f) = \frac{z_0}{z(t)} \frac{R(t_f)}{R_0} \frac{R(\tilde{t})}{\tilde{t}} = \Delta m R(t_f) \quad (43)$$

So, the ratio of the final thickness when no axion will remain inside to the initial thickness becomes,

$$\frac{z(t_f)}{z_0} = \frac{1}{\Delta m} (\tilde{t} t_0)^{(-1/2)} = 8.31 \times 10^{-9} (\frac{t_c}{sec})^{-1/3} (\frac{t_0}{sec})^{-1/2} (\frac{\eta_a}{10^{12} GeV})^{5/6} \quad (44)$$

Since, the time scale (t_c) at the onset of QCD phase transition is of same order of magnitude as our initial time t_0, which is of the order of $\sim 10^{-5} sec$ we get the ratio of final thickness to the initial thickness as $\sim 7.7 \times 10^{-7}$ for string formation scale $\eta_a \sim 10^{10} GeV$. Therefore when the wake inside which axions were trapped reduced to a size of about $\sim 0.1 cm$, the axions which were still remained inside the wake will all leave the wake and form a sheet like region. The release of the axions from the wake thus happens for the duration when wake thickness lies between 2 m and 0.1 cm. Below the thickness of 0.1 cm, no axion will remain inside the wakes. The axions thus left behind as the interfaces collapse will be concentrated in a sheet like regions. One can calculate the density profile of these axions as follows.

Let, $\rho_{pr}(z)$ be the density of axions which is left behind at position z as the interfaces collapse. Then we can write down the following expression which relates total number of axions at position z to the density which are left behind as follows,

$$N(z) - N(z - dz) = A dz \rho_{pr}(z), \quad (45)$$

where the time dependence of z is given in Eq.(29). So We get,

$$\frac{dN}{dz} = A \rho_{pr}(z). \quad (46)$$

Thus we finally get the density profile of axions as a function of thickness with collapsing interfaces as,

$$\rho_{pr}(z) = V_0^{(\frac{-2}{3})} (\frac{R_0}{R(t)})^2 (\frac{dN}{dz}). \quad (47)$$

However, here we should mention that in deriving the above equation we have considered that the axions will not disperse away immediately as it leaves the wake. Strictly speaking, the axions after leaving the wake will move with a velocity corresponding to the momentum

they have in the hadronic phase, as they leave the wake. Since axions leave as soon as their momentum in the QGP phase equals the mass difference Δm (Eq.(1)), their kinetic energy may not be large in the hadronic phase. Hence they may not move very far from the wake (note, axions are very weakly interacting). In this case, the density will be somewhat decreased compare to the density as obtained from the Eq.(47). However the order of magnitude may not be changed much. So for simplicity we will consider the case where the axions will not be homogenized immediately after leaving the wake. In the following section we will discuss the results as is obtained from Eq.(36), Eq.(38) and Eq.(47).

V. RESULT

Eq.(24),(26) are solved numerically to get the evolution of scale factor and volume fraction f which is occupied by the QGP phase. The solution thus obtained is fed into Eq.(38), and Eq. (42) to get the evolution of axion density within the wake as the transition from QGP to hadron phase proceeds. Fig1. shows the evolution of axion density inside the wake for two time zones. Time axis is given in the unit of μsec, while density is in fm^{-3}. As is shown in the figure, most of the time during which transition from QGP phase to hadronic phase ($\Delta t_{trans} \simeq 15\mu sec$) happens, the axions will remain inside the wake. The reason is obviously due to $\frac{1}{k^2}$ dependence of axions density spectrum. The number of axions with minimum momentum will be more and they will leave the over density region last. In Fig.1a, the density keeps increasing solely (for the time zone ($t_0 < t < t_1$)) due to decrease in volume of QGP phase. While for next time zone ($t_1 < t < t_f$), there are two competitive processes. Decrease in volume causes to increase the axion density, while leaking out of axions will cause the density to decrease. Most of the time during which transition from QGP phase to hadron phase proceeds, first effect will be dominated over the latter, hence there will be net increase in density which is evident from the Fig1b. Finally, almost all the axion will acquire the momentum needed to escape from the QGP regions is achieved and density sharply decreases down towards zero. The inset in Fig.1b shows the detailed profile of decreasing part of the axion density plot. Fig2 shows the evolution of density (in unit of fm^{-3}) with the thickness $z(t)$ (in unit of meter). Fig2a, shows the plot upto thickness $z(t_1)$ until which density will keep on increasing due to the effect mentioned above. Fig2b, gives the evolution of density from thickness $z(t_1)$ to $z(t_f)$. For convenience, the inset in Fig2b. shows the expanded plot of the regions where density rapidly goes down towards zero. Fig.3 shows the plot of total number of axions inside the wake as a function of thickness for the time zones, $t_1 \leq t \leq t_f$. Here, volume of a wake is taken as the average thickness of the wake multiplying by the area $A(t)$ (taking 15 long strings per horizon [24] and assuming the sheets extend across the horizon, area of each planar sheet is given as $A(t) \sim \frac{(2t)^2}{15}$) as discussed earlier. The inset in Fig.3 shows the expanded plot within very small distance interval to illustrate the decrease of the total number of axions with the interface motions. Finally, the density profile of axions which are left behind as the interface collapse which is obtained from Eq.(47) is shown in Fig4. Here, plot has been given for the relevant time interval ($t_1 < t < t_f$).

We can take average axion density of the order of $\sim 10^2$ (in Fig.4), then we see that as

the interfaces bordering the wake is reduced to about ~ 0.1 cm, then almost all the axions will leave the wake (see Fig.3) and the density will be increased by a factor of $\sim 10^5$. Since, the energy of the axions required to escape from the wake regions comes from the walls of the interfaces, these are kind of iso-curvature fluctuations. One can also calculate the total mass (M_a) of the axions which are concentrated within this planar sheet region (formed by left behind axions). Taking total number of axions N_a of the order of $\simeq 10^{57}$ (See Fig.3), the mass of the axionic sheet can be obtained from Eq.(7) and given by,

$$M_q \simeq N_a \; m_{ah} \simeq 6.04 \times 10^{-6} eV (\frac{10^{12} GeV}{\eta_a}) N_a \simeq 10^{19} (\frac{10^{12} GeV}{\eta_a}) gm \tag{48}$$

Taking $\eta_a \simeq 10^{10} GeV$, the upper bound on mass of the axionic sheet will be order of 10^{21} gm ($\sim 10^{-12} M_\odot$ at QCD epoch). Since, the fluctuations produced by axion trapping are of iso-curvature kind, they grow little [9,32] before radiation-matter equality era, t_{eq}. After t_{eq}, they grow in proportional to the scale factor. Hogan and Rees [32] have studied the evolution of iso-curvature fluctuations which were produced at the era of QCD phase transition. They have shown that iso-curvature perturbations produced by axions at QCD epoch can lead to formation of axionic 'minicluster'. The amplitude of the iso-curvature fluctuations produced in our model are very large and axions are concentrated within a very narrow sheet like regions of thickness of order 0.1 cm. This sheet will extend to a distance scale of order $\sqrt{A}(t)$. Where, $A(t) \simeq \frac{(2t)^2}{15}$ is the typical area of the wake as discussed above. This is about 2 km at QCD scale, which corresponds to comoving distance scale of order 10^{-7} Mpc today. One can study the evolution of such sheet like overdense region as it enters into the radiation-matter equality era. One can also study these overdensity at larger distance scales resulting from the large scale distribution of strings and their wakes. If these fluctuations survive until late stages, it will be interesting to study the effects of sheet like axionic clusters, especially whether they can have any effects on small scale CMBR anisotropies.

VI. CONCLUSION

In this paper, we have studied the iso-curvature fluctuations in axion density at QCD phase transition epoch due to the presence of density fluctuations produced by moving cosmic strings. We have considered the axions which are produced from the radiation of the axionic strings which are formed at some scale η_a due to breaking $U(1)_{PQ}$ symmetry. If the mass of the axions is relatively higher in the hadron phase compared to QGP phase then the axions may get trapped initially inside the wake-like overdensity regions. As the transition from hadronic phase to QGP phase proceeds with the motion of interfaces, these axions will acquire momentum due to collapsing interfaces and subsequently leave the wake. The axions thus left behind as the interfaces collapse may produce iso-curvature fluctuations. We have estimated the detailed profile of the fluctuations. We have shown that, in our model the iso-curvature fluctuations in the axion density will be of order $(\frac{\delta\rho}{\rho})_{axion} \sim 10^5$ and they will be concentrated within a planar sheet like region of thickness few cm. This sheet can extend upto a distance scale of order 2 km at QCD scale. This length scale corresponds to comoving length scale of order 10^{-7} Mpc today. It will be interesting to study the implications of such

large fluctuations in the axion density especially on small scale CMBR fluctuations. Here, we should mention that, though we have discussed the density structure of wake for straight strings as well as for perturbed strings, results are quoted for straight string wakes only. For wiggly strings, due to the lack of proper relativisitic analysis of shock formation, we have followed the analysis of ref. [22] and quote the resulting density fluctuations and opening angle of the wake (Eq.(16)) for wiggly string. The formation of shock in ref. [22] has been studied for straight string moving through relativistic fluid. In their analysis, the fluid flow has been taken to be uniform. This assumption may not hold for wiggly string. This is because, at the presence of wiggles, the fluid will experience the rapidly changing directions of the wiggles which causes acceleration of the fluid in different direction. So, in proper treatment of shock formation by wiggly string, one should take the non-uniformity nature of the fluid in the analysis. We have also discussed the case where axion (being collisionless) directly (other than trapping mechanism) can give rise to density fluctuations. It turns out that, the value of these fluctuations are very small compared to the fluctuations produced from trapping of axions by cosmic string wakes.

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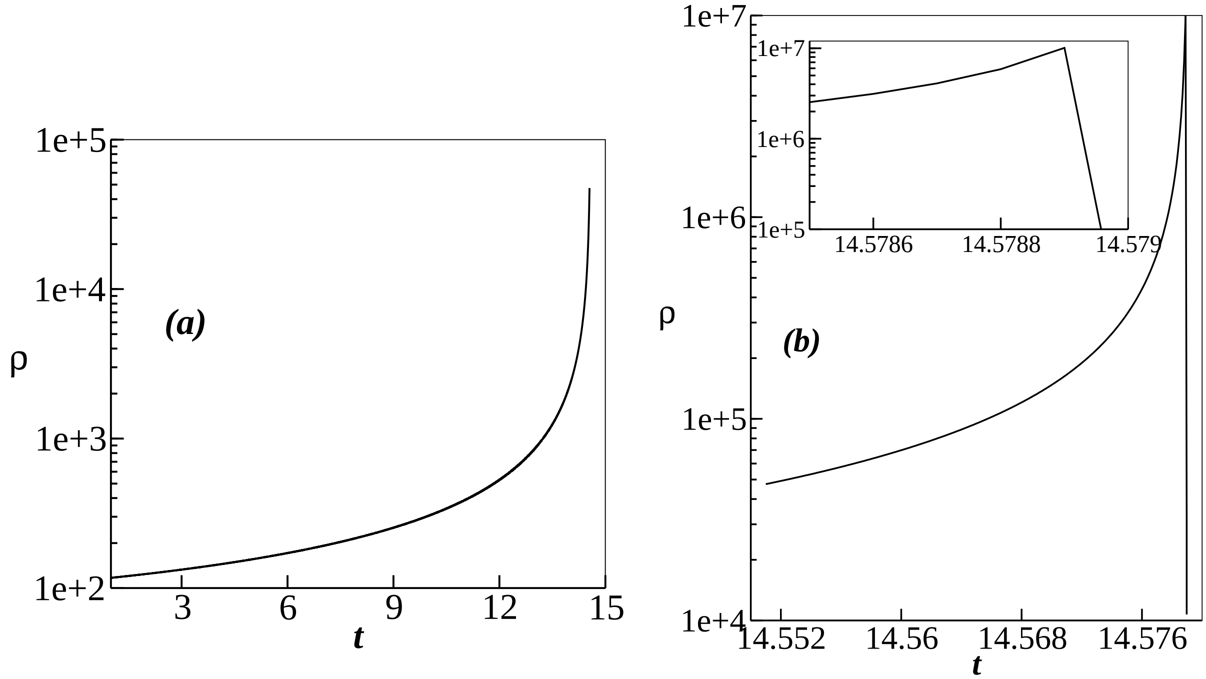

FIG. 1. These figures show plot of evolution of axion density inside the wake as a function of time. The density ρ is given in unit of fm^{-3} and time t is in μsec. Figures (a) and (b) show the density plot for two time zones $t_0 \leq t \leq t_1$ and $t_1 \leq t \leq t_f$ respectively.

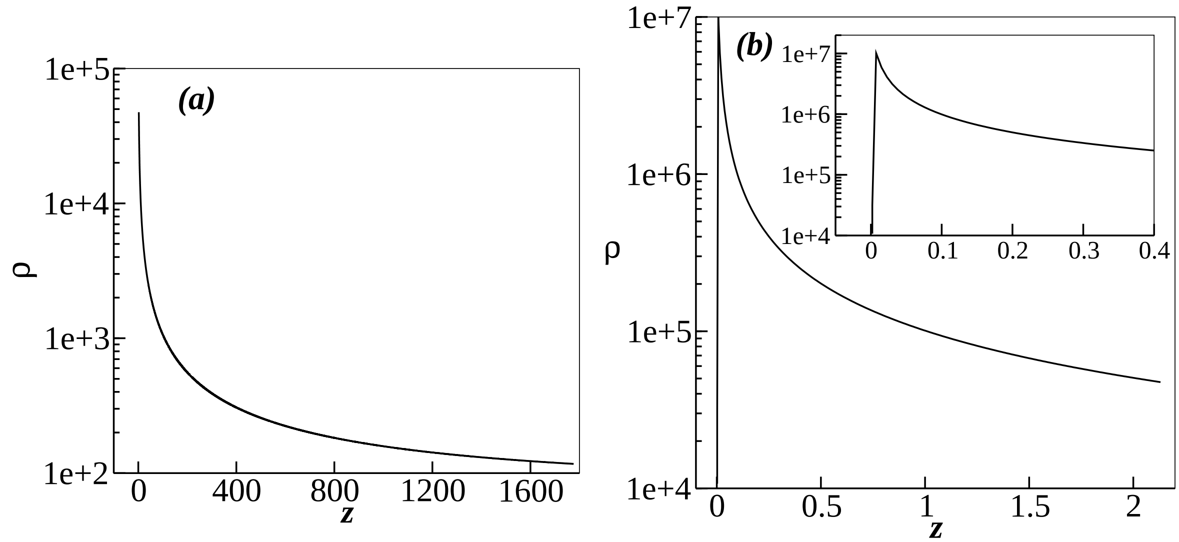

FIG. 2. These figures show plot of evolution of axion density ρ (in unit of fm^{-3}) inside the wake as a function of thickness z (in $meter$) of the wake. Fig.(a) shows the plot upto thickness $z(t_1)$ and Fig.(b) is from $z(t_1)$ upto $z(t_f)$.

FIG. 3. This figure shows the decrease of total number of axions during the time interval $t_1 < t < t_f$ as a function of thickness z (in *meter*). Small part of the apparently flat portion of the plot is expanded in the inset.

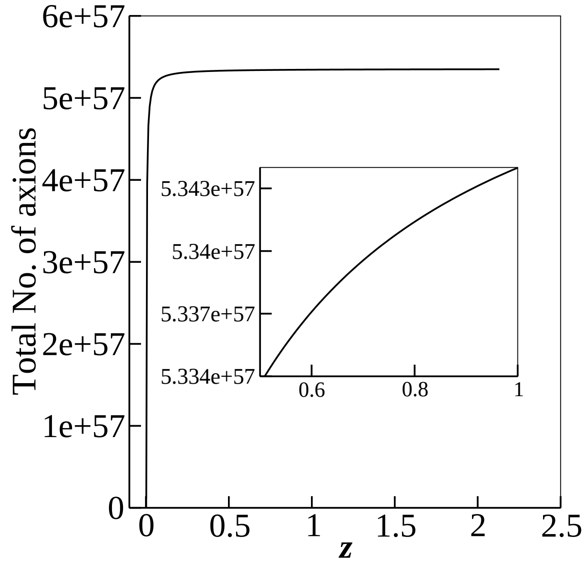

FIG. 4. This figure shows the density profile of the axions which are left behind as the interfaces collapse. Density is given in unit of fm^{-3} and thickness z in $meter$.